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The Tevatron Connection

Theory of $\Delta\Gamma_{B_s}$

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$\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing

Schrödinger equation:

$$i\frac{d}{dt}\begin{pmatrix} |\, B_s(t)\,\rangle \\ |\, \overline{B}_s(t)\,\rangle \end{pmatrix} = \left(M - i\,\frac{\Gamma}{2}\right)\begin{pmatrix} |\, B_s(t)\,\rangle \\ |\, \overline{B}_s(t)\,\rangle \end{pmatrix}$$

where $B_s \sim \overline{b}s$ and $\overline{B}_s \sim b\overline{s}$.

3 physical quantities in $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing:

$$|M_{12}|\,, \quad |\Gamma_{12}|\,, \quad \phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

Two mass eigenstates:

Lighter eigenstate: $|\, B_L\,\rangle = p|\, B_s\,\rangle + q|\, \overline{B}_s\,\rangle.$

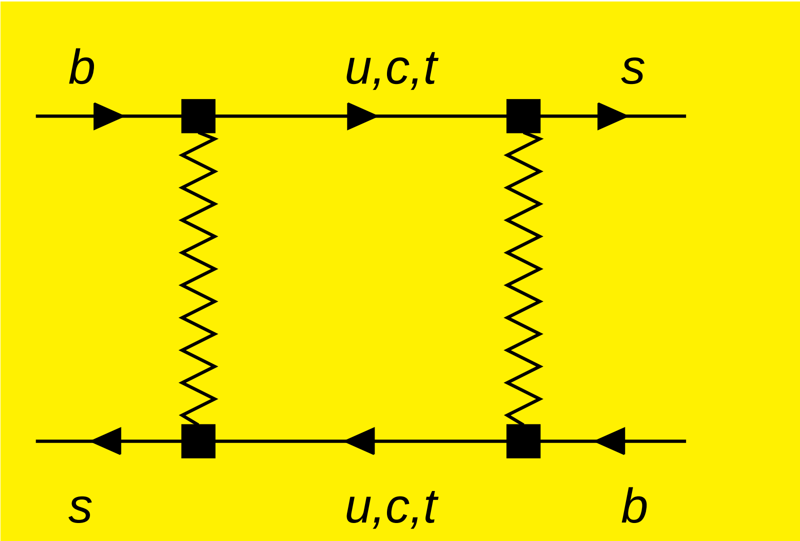
Heavier eigenstate: $|\, B_H\,\rangle = p|\, B_s\,\rangle - q|\, \overline{B}_s\,\rangle$ with $|p|^2 + |q|^2 = 1.$

with masses $M_{L,H}$ and widths $\Gamma_{L,H}$.

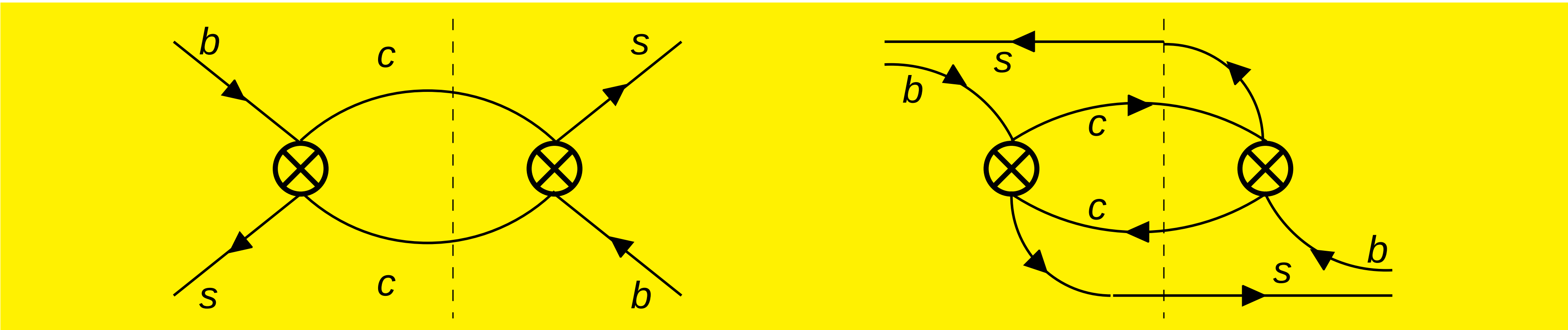
Relation of Δm and $\Delta\Gamma$ to $|M_{12}|$, $|\Gamma_{12}|$ and ϕ:

$$\Delta m = M_H - M_L \simeq 2|M_{12}|, \qquad \Delta\Gamma = \Gamma_L - \Gamma_H \simeq 2|\Gamma_{12}|\cos\phi$$

M_{12} stems from the dispersive (real) part of the box diagram, internal $(\bar{t}, t)$.
Γ_{12} stems from the absorpive (imaginary) part of the box diagram, internal $(\bar{c}, c)$.
(u's are negligible).



Γ_{12} stems from final states common to B_s and $\overline{B}_s$.



Crosses: Effective $|\Delta B| = 1$ operators from W-exchange.

Γ_{12} is a CKM-favored tree-level effect associated with final states containing a $(\overline{c}, c)$ pair.

Theory prediction

$$\Delta\Gamma \;=\; \Gamma_L - \Gamma_H \;\simeq\; 2|\Gamma_{12}|\cos\phi$$

with $\cos\phi \simeq 1$ in the Standard Model.

Corrections to Γ_{12} of order Λ_{QCD}/m_b: Beneke, Buchalla, Dunietz 1996

Corrections to Γ_{12} of order $\alpha_s(m_b)$: Beneke, Buchalla, Greub, Lenz, U.N. 1998
Ciuchini,Franco,Lubicz,Mescia,Tarantino 2003

Prediction (updated to current values of m_b and m_s):

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{210\,\text{MeV}}\right)^2 \left[0.006\,B \;+\; 0.172\,B_S \;-\; 0.063\right]$$
$$= 0.12^{+0.04}_{-0.03}$$

using lattice results for hadronic parameters (Lattice 2004 average):

$$f_{B_s} = 246 \pm 16\,\text{MeV}, \qquad n_f = 2 \text{ and } n_f = 2+1$$
$$B_S = 0.86 \pm 0.07\,\text{MeV}, \qquad n_f = 0$$

With a recent MILC result (hep-ph/0311130):

$$f_{B_s} = 260 \pm 29\,\text{MeV}, \qquad n_f = 2+1$$
$$\Rightarrow \left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = 0.14 \pm 0.05$$

$$\frac{\Delta\Gamma}{\Delta m} = (4.0 \pm 1.6) \times 10^{-3}$$

Beneke,Buchalla,Lenz,U.N. 2003

The CDF experimental value

$$\frac{\Delta\Gamma}{\Gamma} = 0.71^{+0.24}_{-0.28} \pm 0.01 \qquad \text{constrained with } \Gamma_d = \Gamma_s,$$

$$\frac{\Delta\Gamma}{\Gamma} = 0.65^{+0.25}_{-0.33} \pm 0.01 \qquad \text{unconstrained,}$$

is 2.0 σ or 1.5 σ above the central value of the theory prediction.

$\Rightarrow$ nothing to worry about. . . yet.

New physics

Can new physics significantly enhance $\Delta\Gamma$?

Need to increase $|\Gamma_{12}|$ to enhance

$$\Delta\Gamma \;\simeq\; 2|\Gamma_{12}|\cos\phi,$$

but Γ_{12} stems from CKM-favored tree-level decays.

$\Rightarrow$ Any competitive effect from new physics would be seen in $b \to s$ decays of the B^+ or B_d.

But:

The measurement starts to constrain new physics scenarios with $\cos\phi \sim 0$.

Could theory seriously underestimate $\Delta\Gamma$?

The theoretical calculation uses the Heavy Quark Expansion (HQE), which is a power expansion in Λ_{QCD}/m_b. Non-analytical terms like

$$\frac{\sin(-c\, m_b/\Lambda_{QCD})}{m_b^n}$$

are not reproduced.

But the calculation of $\Delta\Gamma$ is very similar to the one of $\tau(B^+)/\tau(B_d)$, which agrees with experiment.

A large $\Delta\Gamma$ implies large B_s branching fractions into final states with quark content $(\overline{c}, c, \overline{s}, s)$ which are CP-even. In the small velocity limit $\Delta\Gamma$ comes from $B_s \to D_s^{+(*)} D_s^{-(*)}$ decays only.

Interesting cross-check:

U-spin symmetry $\Rightarrow$ Study the (Cabibbo-suppressed) B_d decays into $(\overline{c}, c, \overline{d}, d)$ final states at the B factories, in particular $B_d \to D^{+(*)} D^{-(*)}$.

Litmus test of the HQE:

$$\Gamma_d = \Gamma_s \left[1 + \mathcal{O}(1\%)\right]$$

New physics can change this relation by a few $\%$.

Keum,U.N. 1998

Conclusions

Theory predicts:

- $$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{210\,\mathrm{MeV}}\right)^2 \left[0.006\,B \;+\; 0.172\,B_S \;-\; 0.063\right]$$

- $$\frac{\Delta\Gamma}{\Delta m} = (4.0 \pm 1.6) \times 10^{-3}$$

- With increasing statistics the measured central value for $\Delta\Gamma$ will come down.